# 1. LUNAR-ORBIT LANDING SITES AND STAY TIMES

By Robert H. Tolson


## SUMMARY

The determination of possible lunar-landing sites and the corresponding exploration times are discussed for missions in which the lunar-orbit-rendezvous technique is utilized. First, the geometrical properties of lunar orbits, which can be established from typical earth-moon transfer trajectories, are reviewed from the standpoint of the lunar-landing-site problem. Use of a landing procedure which allows mission abort during the entire exploration period indicates that a large region of the moon can be explored for a few days or less. Finally, a brief discussion is given of the possibility of increasing the landing-site capability by varying the parameters in the analysis.


## INTRODUCTION

In the initial Apollo missions the location of the landing site on the lunar surface will probably not be one of the primary parameters determining the over-all mission profile. However, in later missions, landings in particular regions of the moon's surface may be desirable for scientific or other reasons; therefore, it is of interest to determine the lunar sites which are accessible for the Apollo mission and, in particular, to determine the possible lunar-landing sites when the lunar-orbit-rendezvous technique is utilized. The establishment of a lunar orbit is, of course, an integral part of the lunar-orbit-rendezvous technique and, as illustrated in figure 1, the types of lunar orbits that can be established play an important role in determining the possible lunar-landing sites. It is assumed that the mission is initiated with a launch from Cape Canaveral and that after an appropriate coasting phase the injection into the transfer trajectory occurs at an altitude at 480 km with an injection angle of $0^{\circ}$. Typical values of these two parameters are chosen, because it simplifies the analysis and because the results do not sensibly depend on their particular values. At the proper point in the vicinity of the moon an impulse is applied to establish the lunar orbit. The orientation of the resulting orbital plane is specified by the inclination of the orbital plane to the earth-moon plane $i$ and by the longitude of the ascending node of the orbit measured from the earth-moon line $\Omega$.

Although the lunar-landing procedure for the lunar excursion module has not been specified in detail, it is expected that the lunar excursion module will always land nearly in the orbital plane of the command module. Hence, for a given lunar orbit, the possible landing sites constitute a narrow band around the moon directly below the orbit of the command module. The width of this band will depend on the propulsion capability of the lunar excursion module to make orbital plane changes during its descent to the lunar surface. Thus, the location of possible lunar-landing sites is directly related to the geometrical characteristics of lunar orbits which can be established efficiently and which at the same time are consistent with overall mission requirements.

## GEOMETRICAL PROPERTIES OF LUNAR ORBITS

In order to investigate the possible landing sites it is advantageous first to determine the allowable range of $i$ and $\Omega$ consistent with typical constraints on the transfer trajectory. For this purpose, it would be convenient to have explicit relationships between the lunar orbital characteristic and the injection conditions. Unfortunately, no exact expressions are known and some approximations are required in order to obtain general analytical information. For this study the earth and moon are assumed to be spherical masses and to move in circular orbits at their mean distance; in addition, a "patched" conic technique (ref. 1) is utilized. This technique is based on the assumption that there is an imaginary sphere centered at the moon with a radius of about 58,000 km. When the vehicle is inside the "sphere of influence," the earth's gravitational effects on the vehicle are neglected; and when the vehicle is outside of the sphere, the moon's gravitational forces are neglected. With these assumptions it is possible to obtain explicit relationships between the lunar orbital characteristics of interest here and the transfer trajectory injection conditions. Detail of this analysis have been reported in reference 2, but a few of the results are mentioned herein because they have a direct bearing on the lunar-landing-site problem. In figure 2 the sphere of influence is depicted with the moon at its center. The latitude $\eta$ measured from the earth-moon plane and the longitude $\xi$ measured from the earth-moon line are used to specify the location of any point on the sphere of influence. The dashed lines represent the projection onto the sphere of influence of two typical lunar orbits and as before the orientation of these planes will be specified by the inclination and nodal position.

A nominal transfer trajectory is defined as one with a specified injection energy and a specified inclination to the earth-moon plane $I$. A number of free parameters at the injection point are still to be considered and if these free parameters are properly varied, it is in general possible to obtain a wide variet of lunar orbits, namely, lunar orbits with a wide range of values of $i$ and $\Omega$. However, the analysis of reference 2 showed that the orbital planes of all lunar orbits established from a given nominal trajectory pass through a common point o the sphere of influence, that is, all the orbital planes have a common line of intersection. This result provides a relationship between the inclination, noda position, and the latitude and longitude of the common point of intersection of the orbital planes, as follows:

$$\tan i = \frac{\tan \eta}{\sin(\Omega - \xi)} \qquad (1$$

Thus, if the energy and inclination of the transfer trajectory are specified, th values of $\xi$ and $\eta$ are fixed; consequently, the inclination and nodal positio are not independent parameters and once one is chosen the other is determined by this relation. (The importance of this result on the location of possible lunar-landing sites is discussed subsequently.)

This common point of intersection is often called the entry point because this is the point on the sphere of influence through which the vehicle must pass if it is to impact normal to the moon's surface; that is, the extended trajectory would pass through the center of the moon. The utility of the patched conic technique is that the locations of these so-called "entry points" are defined by a set of algebraic equations that can be solved to give the latitude and longitude as a function of the transfer trajectory characteristics. A typical solution of these equations is shown in figure 3. The lower part of the figure gives the locus of entry points for four injection velocity ratios (1.0 corresponding to parabolic velocity) and for a range of transfer trajectory inclinations to the earth-moon plane. At the lower velocities the flight time from the earth to the moon is excessively long and to inject at the higher velocities generally results in a payload penalty, so that it is actually only the middle third of this region which is acceptable for manned missions. In other words, all of the orbital planes must pass through a rather small region on the sphere of influence.

Briefly, what has been shown about the establishment of lunar orbits is that if a nominal earth-moon transfer trajectory is chosen by specifying the injection energy and inclination, then the orbital planes of all lunar orbits established from this nominal trajectory pass through a common point or entry point on the sphere of influence. This result gives a relation between the geometrical properties of the resulting lunar orbits in terms of the latitude and longitude of the entry points. Finally, for manned missions the allowable variation in $\xi$ and $\eta$ as they appear in the relation cannot be changed appreciably by varying the injection conditions of the nominal trajectory.

## LUNAR-LANDING SITES AND EXPLORATION TIMES

With the foregoing relationships established between the lunar orbital characteristics and the earth injection parameters, the allowable stay times on the lunar surface as defined by lunar-orbit-rendezvous requirements may be considered. Some of the parameters in such an analysis are illustrated in figure 4. Suppose a landing at some selected latitude on the lunar surface is desired. One approach to the problem is to establish a lunar orbit with an inclination $i$ greater than the required latitude by the offset, $\delta$. When the excursion module starts its descent to the lunar surface, a small out-of-plane impulse is applied so that the path is along the dashed line and the landing site is $\delta$ degrees out of the orbital plane of the command module. Because of the moon's rotation on its axis and its orbital motion about the earth, there will be a relative motion between the landing site and the orbital plane. In figure 4 the landing site will appear to move to the right along a parallel of latitude at about 13.2° per day. (Note that no perturbations of the command module from pure Keplerian motion are considered here.) After a certain angular travel $\Delta\theta$, the landing site will again have an offset of $\delta$ degrees and the offset is increasing so that the "return to orbit" phase of the mission would be initiated. The exploration period (stay time) on the lunar surface $\Gamma$ is given by

$$\Gamma = \frac{\Delta\theta^{o}}{13.2} \text{ days} \tag{2}$$

where

$$\cos\frac{\Delta\theta}{2} = \frac{\cos i \sin\lambda - \sin\delta}{\cos\lambda \sin i} \tag{3}$$

At any time during the exploration period the lunar excursion module can return to the command module with not more than an offset of $\delta$ degrees required.

Now, the significance of the previously derived relationship between orbital inclination and nodal position is realized. For if $\delta$ is specified by the propulsion capabilities of the excursion module, then the latitude of the landing site determines the required orbital inclination by the relation that Inclination = Latitude + Offset. However, as was shown previously for a specified transfer trajectory, the nodal position is determined once the orbital inclination is chosen. Hence, if this type of landing maneuver is utilized, the landing-site location on any parallel of latitude is uniquely determined by the transfer trajectory characteristics and the offset. Thus, if a maximum stay time is to be obtained for fixed offset, a rather strong restriction on the possible landing sites must be accepted.

The landing-site capability can be increased at the expense of decreasing the allowable stay time on the lunar surface by a slight modification of the landing procedure illustrated in figure 4. Instead of landing at the most extreme point to the left, the landing can be made at some point between the two extremes along the desired parallel of latitude. Again, the exploration time is limited to the time it takes for the landing site to move to the right-hand limit point. With this variation of the original landing procedure there is a range of possible landing sites along each parallel of latitude, and each landing site has a certain stay time associated with it.

The location of the possible landing sites on the lunar surface and the stay time at each site for any nominal earth-moon transfer trajectory can be calculated using equations (1), (2), and (3), together with the results given in figure 3. Figure 5 gives the landing-site stay-time restriction for a median-energy, low-inclination transfer trajectory. The figure shows the face of the moon as seen from the earth. The boundaries are the locus of points on the lunar surface at which landings can be made and at which the lunar excursion module can stay for the specified time without requiring landing and take-off offsets of more than $5^{o}$. The hatched line is the line of maximum stay time corresponding to the landing procedure illustrated in figure 4. In the equatorial and polar regions the excursion module can stay on the surface indefinitely and have the capability of returning to the command module within the limiting amount of offset; for example, if it is possible to establish an equatorial lunar orbit, then the excursion module can land within $5^{o}$ of the lunar equator, and the rotation of the moon on its axis will not affect the relative position of the excursion module with respect to the orbital plane of the command module.

It is seen that a considerable portion of the lunar surface is subject to manned exploration for periods of a few days or less. However, using this landing procedure with the specified offset and the specified nominal transfer trajectory will not allow landings in the midlatitudes on the western limb of the moon. Before the landing capability can be extended into this region, either the offset, the transfer trajectory characteristic, or the landing procedure must be changed.

Increasing the offset will not be considered since this parameter is directly related to the propulsion capabilities of the lunar excursion module, and a sizable increase above the value used here is not expected. In addition, changing the transfer trajectory characteristics will not yield a large increase in the possible landing area because the location of the landing sites on the lunar surface is essentially determined by equation (1), and the results given in figure 3 indicate that for manned missions, the location of the entry points is limited to a small region on the sphere of influence. Thus, the values of $\xi$ and $\eta$ that appear in equation (1) can only take on variations of about $\pm 8^\circ$ from the mean values used to obtain the results of figure 5, and consequently changing the transfer trajectory characteristics would result in a displacement of the boundaries shown in figure 5 by only $\pm 8^\circ$ in both latitude and longitude.

Finally, there are a number of ways in which the landing procedure can be altered so that the excursion module can land at the midlatitudes on the western limb of the moon. First, the landing procedure considered heretofore was designed so that the excursion module could return to the command module at any time during the exploration period without requiring an offset of more than $\delta$. As mission experience is gained, such a requirement may be relaxed in order to increase the landing-site capabilities inasmuch as without this requirement, it is possible to land at any point on the lunar surface. The motion of the moon may take the excursion module a large distance out of the orbital plane of the command module; however, twice during the lunar month the orbital plane will pass over the landing site and the return flight can be initiated. The exploration time at each point on the surface is fixed by simple geometrical constraints and again by the results illustrated in figure 3 and equation (1).

Consider again landing procedures which allow return during the entire exploration period. Note that for a specified landing site on the western limb it is possible to establish a lunar orbit such that after landing, the excursion module will have a displacement of $\delta$ to the west of the orbital plane of the command module; however, the inclination will be greater than the sum of the offset and the latitude. The moon's rotation will again cause the landing site to move eastward relative to the orbital plane; and after a short angular travel, the landing site will be east of the orbital plane with an offset $\delta$ and the offset will be increasing. If the excursion module returns to the command module at this time, the exploration period in days is given by:

$$\Gamma = \frac{1}{13.2}\left[\sin^{-1}\left(\frac{\sin\lambda\cos i + \sin\delta}{\cos\lambda\sin i}\right) - \sin^{-1}\left(\frac{\sin\lambda\cos i - \sin\delta}{\cos\lambda\sin i}\right)\right] \tag{4}$$

This equation gives an exploration time of at least 3.6 hours per degree of offset. Therefore, with this landing procedure and a $5^{\circ}$ offset, every point on the western limb can be explored for at least 18 hours.

Although it has not been stated explicitly, it has been assumed that the lunar excursion module initiates its landing maneuver soon after the lunar orbit is established. Suppose, however, that the excursion module does not land immediately. Because of the moon's motions, the nodal position of the orbital plane of the lunar excursion module will appear to move westward along the lunar equator. If after some specified waiting time in orbit the landing procedure illustrated in figure 4 is initiated, the nodal position will have precessed westward relative to the lunar surface from its original position through an angle equal to the product of the moon's rotational rate and the waiting time in orbit. Therefore, for a specified waiting period, the landing sites and the correspondin exploration times could be obtained from figure 5 by simply rotating the areas in the figure westward through the aforementioned angle. To increase the area of possible landing sites to the entire western limb would require waiting times in orbit of about 6 days.

## CONCLUDING REMARKS

For lunar missions utilizing the lunar-orbit-rendezvous technique, the determination of the possible lunar-landing sites and the corresponding exploration times is seen to depend first on the geometrical properties of lunar orbits which can be established from typical earth-moon transfer trajectories and secondly on the particular lunar-landing procedure utilized. For a landing procedure which affords mission abort during the entire exploration period, a large region of the moon can be explored for periods of a few days or less. A sizable increase in the possible landing-site area on the lunar surface cannot be obtaine by changing the characteristics of the transfer trajectory; however, there are a number of variations in the basic landing procedure which allow landings to be made at any point on the lunar surface.

## REFERENCES

1. Plummer, H. C.: An Introductory Treatise on Dynamical Astronomy. Dover Publ., Inc., 1960.

2. Tolson, Robert H.: Geometrical Characteristics of Lunar Orbits Established From Earth-Moon Trajectories. NASA TN D-1780, 1963.

UNCLASSIFIED

[redacted]

ESTABLISHMENT OF LUNAR ORBIT

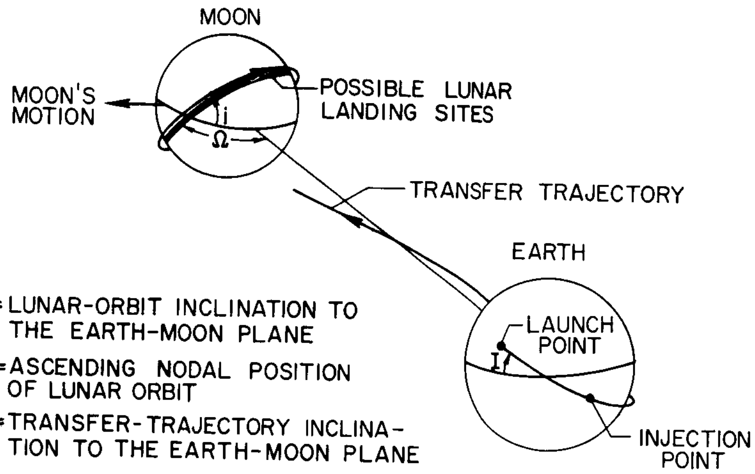


Figure 1

GEOMETRICAL CONSTRAINTS ON LUNAR ORBITS

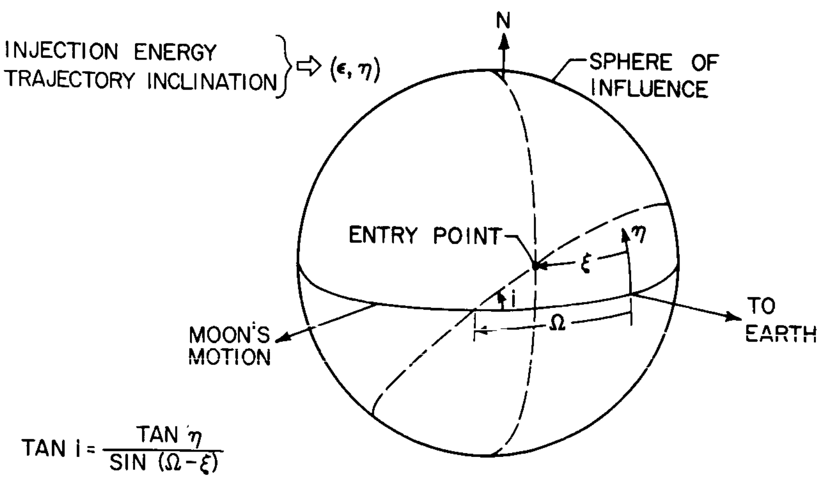


Figure 2

[redacted]



UNCLASSIFIED

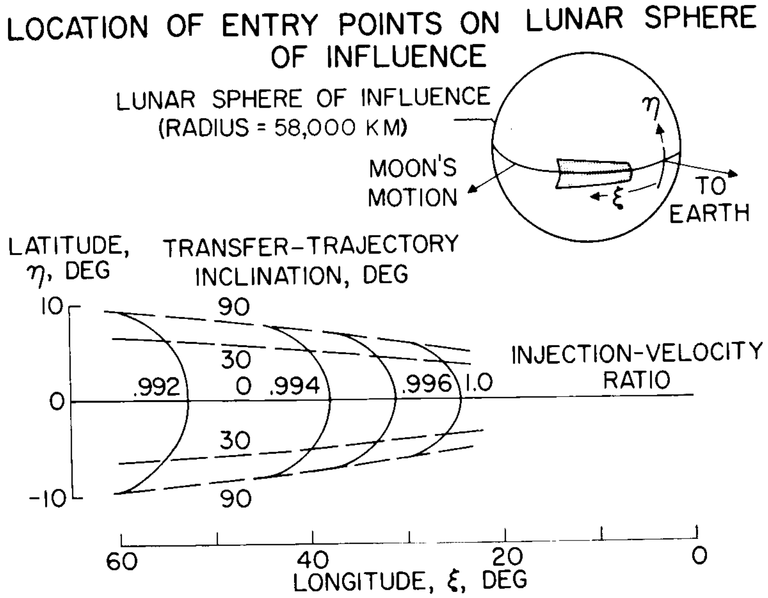


Figure 3

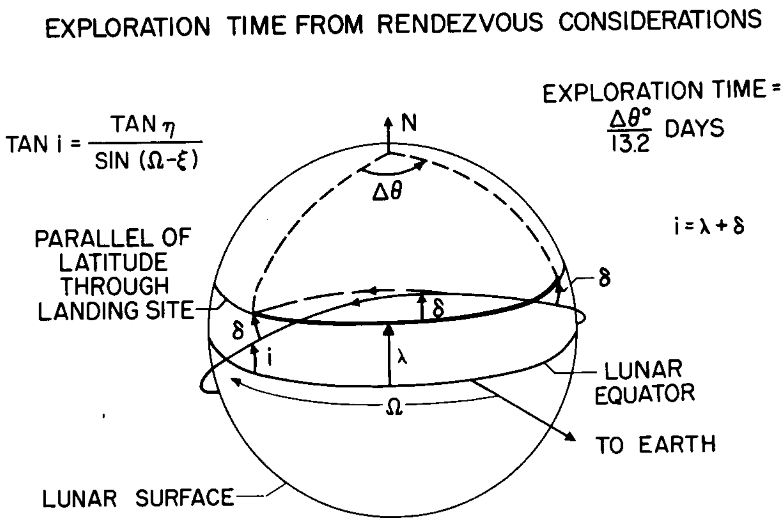


Figure 4

LANDING SITE - STAY TIME RESTRICTIONS FROM RENDEZVOUS CONSIDERATIONS, $\delta = 5°$

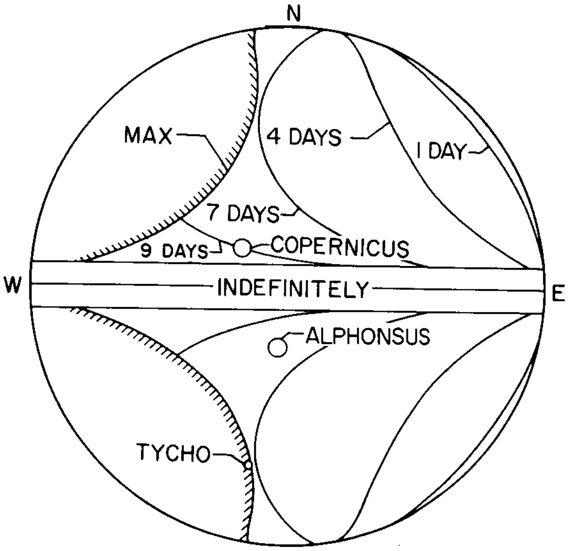


Figure 5